1	Retrieval of Snow Properties for Ku- and Ka-band Dual-Frequency Radar
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# 31 Abstract

The focus of this study is on the estimation of snow microphysical properties and the associated bulk parameters such as snow water content and water equivalent snowfall rate for Ku- and Kaband dual-frequency radar. This is done by exploring a suitable scattering model and the proper particle size distribution (PSD) assumption that accurately represent, in the electromagnetic domain, the micro/macro-physical properties of snow. The scattering databases computed from simulated aggregates for small to moderate particle sizes are combined with a simple scattering model for large particle sizes to characterize snow scattering properties over the full range of particle sizes. With use of the single scattering results, the snow retrieval look-up tables can be formed in a way that directly links the Ku- and Ka-band radar reflectivities to snow water content and equivalent snowfall rate without use of the derived PSD parameters. A sensitivity study of the retrieval results to the PSD and scattering models is performed to better understand the dual-wavelength retrieval uncertainties. To aid in the development of the Ku- and Ka-band dual-wavelength radar technique and to further evaluate its performance, self-consistency tests are conducted using measurements of the snow PSD and fall velocity acquired from the Snow Video Imager/Particle Image Probe (SVI/PIP) during the winter of 2014 in the NASA Wallops flight facility site in Wallops Island, Virginia.

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#### 1 Introduction

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The Global Precipitation Measurement (GPM) core satellite, a joint Earth-observing mission between the National Aeronautics and Space Administration (NASA) and the Japan Aerospace Exploration Agency (JAXA), was successfully launched into orbit on 27 February of 2014 from Japan (Hou et al. 2008 and 2014). One of the goals of the Dual-frequency Precipitation Radar (DPR) aboard the GPM satellite is to provide measurements and estimates of snow precipitation rate and water content for mid- and high-latitude regions. This is usually done by estimating parameters of snow particle size distribution (PSD) that are often modeled by an analytical function, such as the exponential, gamma or lognormal distribution, with two or three unknown parameters (Gorgucci et al. 2000 and 2002; Bringi et al. 2002). The inability of the modeled PSD to represent actual snow spectra and to characterize their intrinsic variations in time and space can lead to errors in the estimates of precipitation rate obtained from the DPR. Additionally, uncertainties associated with scattering computations of snow aggregates also affect the accuracy of the dual-wavelength radar retrieval of snow arising from the complex shape and structure of snow aggregates and the corresponding variability in the scattering parameters. Therefore, understanding the uncertainties in snow precipitation estimation that depend on PSD parameterizations and scattering models of individual particles is important in evaluating the overall performance of DPR retrieval algorithms and in gaining insight into ways to improve the algorithms.

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Several studies have been carried out using dual-frequency radar for the retrieval of precipitating ice/snow parameters from the ground (Matrosov 1998; Szyrmer and Zawadzki 2014) and from airborne radar measurements (Liao et al. 2005, 2008; Heymsfield et al. 2005; Wang et al. 2005;

Matrosov et al. 2005). Although various combinations of frequencies were used in these studies, a common feature is that at least one of the radar frequencies operates in the non-Rayleigh regime to ensure a measurable difference in the reflectivities. It is this differential reflectivity that can be related to a characteristic size parameter of the snow particle distribution. Because of uncertainties in the snow microphysics arising from the natural variability of the particle density, shape, and orientation and also because of uncertainties associated with the particle backscattering cross section and terminal fall velocity as well as the natural variability in PSDs, it is important to assess the errors in the model and their impacts on the retrievals.

The emphasis of this study is on the estimation of snow microphysical properties and the associated bulk parameters such as snow water content and water equivalent snowfall rate. As indicated earlier, one of the challenges in the radar retrieval of snow is to characterize the variability of the snow PSD and to efficiently compute scattering properties of the snowflakes over the full range of sizes. The aim of our study is to explore a suitable scattering model and an appropriate PSD that accurately represents, in the electromagnetic domain, the micro/macro-physical properties of snow.

Although several scattering databases are available, which provide the scattering properties of snow aggregates (Liu 2004 and 2008; Nowell et al. 2013; Kuo et al. 2016), they are often limited to small-to-moderate particle sizes for a limited set of frequencies. These limitations arise not only because of the magnitude of the computational burden but also because of the convergence properties of the numerical solution. To develop an operational-type radar algorithm for the DPR snow retrieval, it is desirable to have a scattering model that provides efficient computation at an

arbitrary frequency over a large range of particle sizes. Comparisons of the scattering results between simple and more complicated snow models indicate that the scattering properties of aggregates at the DPR frequencies are fairly well reproduced by randomly-oriented ellipsoidal particles if the effective mass density of snow is constant with size (Liao et al. 2013). By taking advantage of both developed scattering databases and simple scattering models, we attempt to employ the scattering results of the aggregates from the scattering database for small to moderate particle sizes and use the results from the simple scattering models for large particles to cover the full range of particle sizes for characterizing snow scattering properties at Ku- and Ka-band.

One of the DPR algorithms for snow retrieval employs a fixed-snow-density spherical model for computations of the Ku- and Ka-band radar backscattering and extinction cross sections using the assumption of an exponential PSD (Seto et al. 2013). To improve snow retrieval accuracy, we will investigate retrieval uncertainties associated with the PSD and the particle scattering models, and introduce new forms of the retrieval look-up tables that directly link DPR Ku- and Ka-band radar reflectivities to snow water content and snowfall rate without the use of derived PSD parameters. Newly formed look-up tables provide an effective means not only for snow retrieval but for analysis of the retrieval uncertainties associated with the PSD model and the particle scattering models. In order to check the consistency of the snow retrievals, measurements of the snow PSD and fall velocity acquired from the Snow Video Imager/Particle Image Probe (SVI/PIP) are used (Newman et al. 2009). It is believed that a collection of long-term PSD data, fall velocities and information on particle mass spectra will provide a strong basis for evaluating the performance of the DPR Ku- and Ka-band techniques.

This article is organized as follows. Methods for snow retrieval are described in Section 2, and measurements of the PSD are discussed in Section 3, followed by remarks and a summary in Section 4.

# 2 Technical Approach and Methodology

Understanding the retrieval errors associated with the snow particle size/mass distributions models and particle scattering models employed by the DPR algorithms is important for the evaluation of algorithm performance. The study also provides insight into which of the models yield the most accurate DPR estimates of snow. Proper selection of the PSD and scattering models can improve the overall performance of the DPR profiling algorithm. What follows are discussions of snow scattering models and the parameterization of the particle size spectra and how these models affect the radar retrievals.

### 2.1 Single scattering model of snow aggregates

Several simulated aggregate models have been developed using the pristine ice crystal habits found in nature as the basic elements from which the aggregates are constructed (Draine and Flatau 1994; Liu 2004 and 2008; Weinman and Kim 2007; Petty and Huang 2010; Botta et al. 2010 and 2011; Nowell et al. 2013; Kuo et al. 2016). For these particles, a numerical scheme is required to compute the scattered fields. Although these numerical computations are useful, they are time consuming, and are often limited to small-to-moderate particle sizes for a limited set of frequencies. A few scattering databases derived from simulated aggregates are available (Nowell et al. 2013; Kuo et al. 2016), but the maximum equivalent ice diameter is limited to around 2.5-3 mm which is not large enough to cover entire particle size range.

To account for the scattering contribution from the entire particle size range, the current DPR algorithms adopt a simple scattering model, namely, the ice-air mixed spheres with a fixed snow density of 0.1 g/cm<sup>3</sup> for all particle sizes (Seto et al. 2013). To check the validity and accuracy of the simple geometric scattering model, a study was carried out by Liao et al. (2013) in which scattering results from aggregates comprised of 6-branch bullet rosette crystals were compared with those obtained from spherical or spheroidal ice-air mixed phase particles. Shown in Fig.1 is an example of these results, in which backscattering (left) and extinction (right) coefficients at 35.6 GHz are plotted versus the equivalent ice diameter for 3 simulated snow aggregates and results from the sphere, oblate and prolate spheroids. A constant effective snow density of 0.2 g/cm<sup>3</sup> is assumed for all spherical and spheroidal particle models. The spheroids are assumed to be randomly oriented, i.e., their symmetry axes are uniformly distributed in three-dimensional space. The aspect ratios  $(\gamma)$  of the spheroids, defined as the ratio of polar to equatorial lengths, are taken to be either 0.5 or 2 to represent oblate and prolate spheroids respectively. The results of the study show that the scattering properties of the aggregates are fairly well reproduced by an equivalent-mass spheroidal particle when a constant snow density is assumed.

Agreement between the spheroidal/fixed density model and the aggregates suggests the validity of the simple model and its utility for computing look-up tables for the DPR. An effective snow density of 0.2 g/cm<sup>3</sup> is best able to reproduce the scattering parameters of the bullet-rosette aggregates at the DPR frequencies. It is important to note that the effective mass density, as defined in this paper, is that mass density of a spheroidal particle whose associated scattering parameters provide the best match to those of the simulated aggregates with the same mass. This definition is motivated by the desire to match the scattering properties of the spheroidal and

simulated aggregates and is distinct from the usual definition of snow density given by the ratio of the particle mass to the volume where the volume is taken to be that of a circumscribing sphere or spheroid. To distinguish our definition from others, the density we use for scattering computations is referred to as the effective snow density. It is also important to note that the snow mass is preserved in both definitions; that is, the product of the mass density or effective mass density and the particle volume yields the same mass.

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Recently, Kuo et al. (2016) have developed a comprehensive scattering database, which is computed using the discrete dipole approximation (DDA) from a collection of realistic aggregates simulated from a 3-D growth model with mass vs. size and fractal properties that are consistent with field observations (Gravner and Griffeath 2009). Because of its limited range of particle size mentioned earlier, the scattering tables of snow aggregates in our study will be taken as a hybrid form that combines the scattering results from the Kuo et al. scattering database for small to moderate particle sizes and the results from the simple scattering models for large particles. Illustrated in Figs. 2 and 3 are these scattering results at Ku- and Ka-bands from the Kuo et al. database along with the results from an oblate spheroidal model for particle diameters up to 6 mm. The results from the simple models (red curves) are those obtained from the randomly-oriented oblate spheroids with an aspect ratio of 0.7 and a constant effective mass density of 0.2 g/cm<sup>3</sup>. The results of the scattering tables, which are denoted by the term "fitted" and shown by the black curves, represent the mean values of the data from the scattering database, where the mean is taken from all types of aggregates having the same mass, and the results from the 0.2 g/cm<sup>3</sup> oblate spheroid model in the size range where the scattering database is unavailable. The maximum liquid equivalent diameters of the current Kuo et al. database at

Ku- and Ka-bands are 3 and 2 mm, respectively. In other words, the scattering results from the simple models are employed in the tables for the size ranges from 3-6 mm at Ku-band and 2-6 mm at Ka-band. The fairly good agreement of the scattering results in Figs. 2 and 3 between the simple model and the scattering database (with mean differences less than 10% for the backscattering and 7% for extinction cross sections) over the size range where the database is valid suggests the validity of the simple models for the smaller particle sizes. As the scattering database is updated to cover larger particle sizes, the validity of the simple scattering model will be reassessed.

## 2.2 Particle size/mass distribution model

The three-parameter gamma distribution is one of the most common ways to mathematically describe hydrometeor size/mass distributions (Gorgucci et al. 2000 and 2002; Bringi et al. 2002).

The form of the gamma distribution is expressed as

$$N(D) = N_W f(\mu) \left(\frac{D}{D_m}\right)^{\mu} \exp(-\Lambda D), \tag{1}$$

where  $D_m$  is the mass-weighted diameter of the particle,  $N_w$  is a scale factor, and  $\mu$  is the shape factor where

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$$f(\mu) = \frac{6(4+\mu)^{\mu+4}}{4^4\Gamma(\mu+4)}$$
 and  $\Lambda = (4+\mu)/D_m$ , (2)

To describe snow particle size and mass spectra, the PSD in (1) and (2) is given as a function of liquid equivalent or melted diameter D, which is also called particle melted-size distribution.  $D_m$  is the melted median mass diameter, defined by

$$D_{m} = \frac{\int_{0}^{\infty} D^{4}N(D)dD}{\int_{0}^{\infty} D^{3}N(D)dD}.$$
 (3)

In the inner swath, the DPR provides Ku- and Ka-band reflectivity factor measurements at each range gate so only two parameters of the PSD can be determined. Typically, the shape factor ( $\mu$ ) is taken to be constant. Although  $\mu$  is often set to zero (exponential distribution) (Gunn and Marshall 1958; Seto et al. 2013), the impact of this choice on the retrieval needs to be investigated.

The differential frequency ratio (DFR), which is defined as the difference between the radar reflectivity factors at Ku- and Ka-bands in decibels, is perhaps the most important quantity for the dual-wavelength radar techniques in estimating hydrometeor micro/macro-physical properties. As the DFR is independent of  $N_w$ ,  $D_m$  can be derived from the DFR relations once  $\mu$  has been fixed. However, the DFR- $D_m$  relation depends not only on  $\mu$  but on the particle shape, orientation distribution and mass density. Fig.4 provides the results of DFR as a function of the liquid equivalent median mass diameter using a randomly-oriented, fixed density spheroidal particle model. The left plot shows the variations in the DFR- $D_m$  relation resulting from different effective snow densities. Computations of the radar scattering parameters at different effective snow densities are made in the same way as in the case of 0.2 g/cm³. The particle sizes (semi axes of spheroid) are solely determined by the density specified for a given particle mass. The center plot shows the effects of particle shape where a  $\gamma$  value of 1 corresponds to a sphere while  $\gamma$  values less than 1 correspond to an oblate spheroid. The plot on the right shows the effect of changing  $\mu$ . Analysis of these results indicates that particle shape has a small effect on

the DFR- $D_m$  relation and changes the results by less than 4% for changes in  $\gamma$  from 0.5 to 1 while the shape factor leads to the change in the results of no more than 20%. On the other hand, the DFR- $D_m$  relation has a strong dependence on the effective snow density. In other words, the determination of  $D_m$  from the DFR is relatively insensitive to  $\mu$  and to particle shape, if the orientation is random, whereas the relationship is quite sensitive to the effective snow density used for computations of the scattering parameters. As can be seen in Figs.1-3, the extinction coefficients at both Ku- and Ka-bands, though in good agreement between the simple model and the scattering database, are small and can usually be neglected. It should be noted that the above conclusions are based on the assumption of random orientation of the aggregates. When this assumption is violated, then the orientation distribution as well as particle shape become important.

Although an effective snow density of 0.2 g/cm<sup>3</sup> is found to be suitable for the computations of the Ku- and Ka-band radar scattering parameters for equivalent ice diameters up to 2-3 mm, further testing will be necessary to assess this assumption when scattering results from larger aggregates become available. It is worth noting that the results of Liao et al. (2013) show reasonably good agreement between radar scattering parameters at higher frequencies (from 89-183.31 GHz) as derived from the simple models and the simulated aggregates for particle diameters up to 2.5 mm despite the fact that the simple models using spheres or nearly spherical particles produce backscattering results with more pronounced oscillations (resonance effects) than the aggregate results. This is encouraging in the sense that it shows that an effective density of 0.2 g/cm<sup>3</sup> yields good agreement with the simulated aggregate results for electrically larger particles. On the other hand, it might be the case that the effective snow density may need to be

changed for larger particle types so as to ensure good agreement. In either case, the objective is to provide scattering tables at all relevant frequencies and particle sizes that incorporate the most recent scattering results.

# 2.3 Dual-wavelength retrieval algorithm

As discussed above,  $D_m$  can be derived from the DFR- $D_m$  relations for a given  $\mu$ . In principle, once  $D_m$  is determined,  $N_w$  is derived using the radar reflectivity at either Ku- or Ka-band. Subsequently, snow water content (SWC) and equivalent snowfall rate ( $R_S$ ) can be computed from the derived PSD parameters. The fall velocity of snow is needed in order to estimate  $R_S$ . For the development of an effective dual-wavelength radar retrieval technique, it is desirable to employ look-up tables (LUT) that are formed in such a way that the radar measurements are directly linked to the microphysical properties of snow ( $D_m$  and  $N_w$ ) and its associated bulk parameters (SWC and  $R_S$ ). With use of the LUTs different particle models and their scattering properties can be evaluated separately in the context of the same algorithm.

Illustrated in Fig.5 are such tables in which SWC (top-left),  $R_{\rm S}$  (top-right),  $D_{\rm m}$  (bottom-left) and  $N_{\rm w}$  (bottom-right) along the ordinate are given as a function of the DFR. A flowchart is provided in Fig.6 showing the procedures to compute the radar reflectivitity factors and snow size and bulk hydrometeor parameters from an assumed mass spectrum model. In Fig.5, SWC,  $R_{\rm S}$ ,  $N_{\rm w}$  have been normalized by the Ku-band radar reflectivity factor so that they can be expressed solely as a function of DFR for given PSD and scattering models. The way to normalize liquid water content by reflectitities has previously been adopted in the study of ice clouds (Hogan et

al. 2000; Botta et al. 2013). The results in Fig.5 are computed under the assumption that the snow particles are fixed-density, randomly-oriented oblate spheroids with an aspect ratio of 0.7 that follow an exponential particle size distribution. As an example, and also for reference, the tables are plotted in Fig.5 for effective snow densities varying from 0.05 to 0.5 g/cm<sup>3</sup>. The terminal velocities of snowflakes used for the computations of R<sub>S</sub> are based on the results of Magono and Nakamura (1965). It is important to note that the results from the LUTs shown in Fig.5 can be used to determine SWC and R<sub>S</sub> as they directly link the DPR radar reflectivities to SWC and R<sub>S</sub> without use of the derived PSD parameters.

The procedure for the estimation of snow parameters is described as follows: given a pair of reflectivity factors ( $Z_{Ku}$ ,  $Z_{Ka}$ ), the DFR in dB is defined as  $10Log_{10}(Z_{Ku}/Z_{Ka})$ , from which we find the values of SWC/ $Z_{Ku}$  (left) and  $R_S/Z_{Ku}$  (right) for an assumed effective snow density. By multiplying by  $Z_{Ku}$ , the results of SWC and  $R_S$  are then obtained. Obviously the values of SWC and  $R_S$  depend on the effective snow density. The estimates of  $D_m$  and  $N_w$  can be achieved in a similar way. It is worth mentioning again that snow attenuations, though correctable, are typically negligibly small for most Ku- and Ka-band spaceborne radar measurements.

As LUTs change with different scattering models and PSD parameterizations, a proper selection of the tables is critical to the accuracy of the retrieval. It is instructive to conduct a sensitivity study with respect to the model assumptions and to gain an understanding of the uncertainties associated with each of the models. Figure 7 provides such a sensitivity study in which the LUTs are checked against 3  $\mu$  values. Similar to the findings in Fig.4, a change in  $\mu$  leads to changes in the estimates of SWC and  $R_S$  of less than 20% so that the assumption of  $\mu$  equal to zero, as found

in many observations, yields a reasonable approximation for the estimates of snow. Although it is worth testing the scattering databases of the aggregates formed from various crystal habits, it is not the focus of the study to evaluate and validate these scattering databases. Because the mass of the aggregates is the dominant factor in the scattering parameters at Ku- and Ka-bands, significant differences among the various scattering databases are not expected. This is evidenced by the fact of that there is good agreement between the scattering databases derived by Nowell et al. (2013) and Kuo et al. (2016) and that the small variations of the scattering and extinction coefficients as computed from various aggregate models (Kuo et al. 2016) can be seen from the variations in the data (blue) shown in Figs.2-3.

### 3 Assessment of snow retrieval: PSD model assumptions

Because of the complexity of snowfall processes and the difficulties encountered in accurately measuring the microphysical properties, validating snow estimates is a challenging task. With the advent of more advanced digital cameras and image processing technologies, measurements of falling snow have been improved to the point where the snow particle size spectra and fall velocities can be obtained fairly accurately (Bohm 1989; Huang et al. 2010 and 2015; Garrett et al. 2012). An independent and direct measurement of the mass of individual snow particles is, however, still a difficult task, and therefore direct measurements of the snow mass spectrum are rarely available. Several investigations into deriving snow mass spectra are being pursued, which are in fact part of the effort in the GPM ground validation project. These methods are based on the principle that particle masses can be related to their fall velocities after accounting for air drag and other aerodynamics effects (Bohm 1989; Heymsfield et al. 2010). Understanding the microphysical properties of snow should further improve our ability to generate better scattering

representations and more accurate look-up tables for retrieving snow bulk properties from the DPR. It would be ideal to evaluate snow retrievals with co-located dual-wavelength radar measurements and in situ snow microphysical measurements. These data, though desirable, are not available. Our attention is therefore focused on the assessment of the PSD assumptions used in developing the retrieval algorithms using measured PSD. To check the consistency of snow retrievals using the LUTs, measurements of snow spectra are used. The data were obtained from 8 snow events during winter of 2014 taken at the NASA Wallops Flight Facility in Wallops Island, Virginia using the Snow Video Imager/Particle Image Probe (SVI/PIP). Table 1 provides details of these events that include starting and ending times of snowfall, mean temperature as well as total accumulations of each event. In Wallops Island annual mean snowfall is about 200.66 mm, and in 2014 it was recorded to be 223.66 mm, slightly more than average. Although the PIP measures the dimensions or sizes of the snowflakes and their fall velocities, it does not provide measurements of particle mass. In order to compute the radar reflectivities and snow bulk parameters as in (4) and (5), the mass spectra or melted size spectra are needed. Conversion of the PSD measurements to the mass spectra, however, relies on the empirical mass-size relations. There are many such relationships available in the literature that can be used to derive m(D) (Nakaya 1954; Magono and Nakamura 1965; Zikmunda and Vali 1972; Locatelli and Hobbs 1974; Mitchell et al. 1990; Brandes et al. 2007; Heymsfield et al. 2010). These results show some variability depending on snow type, amount of riming and other conditions under which the measurements were made. In this study, two wellknown mass-size relations, the results from Heymsfield et al. (2010) and Brandes et al. (2007), are used to test how the estimates of snow change with use of different mass-size relations when the same LUTs are used.

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An example of the PSD measurements is shown in Fig.8, in which N(D) of the PSD (top), shown along the vertical with the amplitude of the spectrum given by the color scale, is given as a function of time. The equivalent snowfall rate (middle) and median mass diameter  $(D_m)$  (bottom) are also shown for the same time period. For the computations of snowfall rate and  $D_m$ , empirical snow mass-size relations are used in conjunction with the measured snow particle size spectra and fall velocities. The following equations are used for obtaining  $R_S$  and  $D_m$ :

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$$D_{m} = \frac{\int_{0}^{\infty} D \ m(D)N(D)dD}{\int_{0}^{\infty} m(D)N(D)dD}, \tag{4}$$

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$$R_S = \frac{36 \times 10^{-4}}{\rho_w} \int_{D_{min}}^{D_{max}} N(D) \, m(D) V(D) dD, \tag{5}$$

where D and  $D_m$  in (4) and (5) as well as in Fig.8 are, respectively, the actual particle diameter and median mass diameter rather than the melted sizes employed in the rest of the paper. m(D) is the particle mass-size relation, and  $\rho_w$  is the water mass density taken to be 1 g/cm<sup>3</sup>. Note also that the data shown in Fig. 8 represent a measurement period of 1000 minutes of snow data (~three snow events) with a one-minute integration time. The mass-size relation of Heymsfield et al. (2010) is used for computations of  $R_S$  and  $D_m$ .

Illustrated in Fig.9 are the scatter plots (red dots) of SWC (top row) and R<sub>S</sub> (bottom row) computed from the measured PSD with use of mass-size relations when the hybrid scattering tables are assumed. For reference, the LUTs derived from the constant effective density scattering models as shown in Fig.6 are superimposed in Fig.9. The SWC and R<sub>S</sub> as well as the quantities associated with the Ku- and Ka-band radar reflectivities shown in Fig.9 are obtained from a total of about 8000 1-minute PSD measurements collected from the PIP during the winter

of 2014 at Wallops using the mass-size relations of Brandes et al. (left column) and Heymsfield et al. (right column). Mass-size relations are used to convert the measured particle size distribution (PSD) to the melted size distribution from which the scatterings and snow bulk parameters can be computed as in (2)-(5). The mass-size relations used in Fig. 9 are those from Eq.(8) of Brandes et al. (2007) and Eq.(10) of Heymsfield et al. (2010). The procedures used in obtaining the radar parameters and SWC and R<sub>S</sub> from the measured PSD are shown in the flow diagram of Fig. 10. Analysis of the SWC results indicates that the snow water content derived from the measured PSD agree reasonably well with those from the tables when the effective snow density is taken to be 0.2 g/cm<sup>3</sup>. Because the scattering table, which is a hybrid formed from the mean aggregate solution at small particle sizes with the spheroidal particle model at larger sizes, used for computing the reflectivities of the measured PSD and the scattering results with a density of 0.2 g/cm<sup>3</sup> are nearly the same as shown in Figs.2-3, the differences in snow water content between the measured PSD and the table results from a density of 0.2 g/cm<sup>3</sup> are mostly caused by the differences between measured and modeled melted particle size distributions. As noted earlier, the exponential melted-size distribution is assumed in the look-up tables while the measured melted-size distribution is derived from the measured particle diameter spectrum and the mass-size relation that generally will be different from an exponential distribution. Computation of the snowfall rate, on the other hand, depends not only on the particle mass (or melted-size) spectrum but also on the particle fall velocities. Most of the estimated snowfall rates, as derived from the measured PSD and the mass-size relations, and shown in the lower panels of Fig.9, lie between the table results with effective densities of 0.1 and 0.2 g/cm<sup>3</sup>. The mean differences of the SWC between the 0.2 g/cm<sup>3</sup> snow density LUTs and the PSD-derived results are about 20% for Ku-band radar reflectivities greater than 15 dBZ (the

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approximate minimum detectable signal of the DPR Ku-band channel). Larger differences between the R<sub>S</sub> estimates are found, and can be attributed to differences in the snow fall velocity spectra between the measured and modeled mass distribution. The terminal velocities of snowflakes used for the computations of the LUTs are based on the results of Magono and Nakamura (1965) while the measured fall velocities are used for the computations of PSD snowfall rate.

The overall agreement of the snow water content between the results from the measured PSD and the results from the LUTs suggests that the exponential particle distribution model assumed in the tables is reasonable. Different mass-size relations lead to different mass spectra for a given measured PSD. That the retrieval results from the Brandes et al. and Heymsfield et al. mass-size relations follow the trends of SWC and  $R_S$  similar to those derived from the LUTs further suggest that the Ku- and Ka-band dual-wavelength techniques adopted are relatively insensitive to the choice of either the Brandes or the Heymsfield mass-size relation. These results approximately yield the table values obtained from the  $0.2~g/cm^3$  snow density. It is also worth mentioning that the results from the measured PSD are relatively insensitive to PSD integration time even though the scatter in the data is slightly reduced if a longer integration is used.

# 4 Summary

The ultimate goal of this study is to better understand the estimation process in retrieving snow microphysical properties ( $N_w$  and  $D_m$ ) and the associated bulk parameters (SWC and  $R_S$ ) for improvement of the Ku- and Ka-band dual frequency radar retrieval. This is done by first finding suitable single scattering tables and PSD models and then using this information to construct

snow retrieval look-up tables. Presently available scattering databases, though accurate and useful, are limited to small and moderate particle sizes. To extend the results to larger sizes, a simple scattering model that agrees well with the scattering databases at small particle sizes is used. It is found that a snow particle model consisting of randomly oriented oblate spheroids with an effective mass density of 0.2 g/cm<sup>3</sup> yields good agreement with the results from the scattering databases at Ku- and Ka-band. Thus the single-particle scattering database is a hybrid that uses the scattering database for small and moderate particles and a simple randomly oriented oblate with a constant effective mass density of 0.2 g/cm<sup>3</sup> for large particles.

Using single scattering tables and an assumed PSD model, the Ku- and Ka-band radar reflectivity factors and snow bulk parameters are computed. Thus, the relationships between the results of DFR and SWC and R<sub>S</sub> are established to form the dual-wavelength radar retrieval look-up tables. Retrievals of snow water content and snowfall rate, as the primary focus of this study, are therefore achieved by using newly introduced look-up tables that directly link Ku- and Ka-band radar reflectivities to hydrometeor parameters without the use of derived PSD parameters. The look-up tables are formed so that SWC and R<sub>S</sub>, both of which are normalized by the Ku-band radar reflectivity factor, are expressed as a function of the differential frequency ratio of Ku- and Ka-bands. The look-up tables offer not only computational advantages but provide direct insight into how the model assumptions impact the retrieval results. The nature of one-to-one relations between the normalized hydrometeor parameters and DFR provides a means to obtain unique solutions of the snow parameters for a given PSD and single scattering model. To understand the uncertainties in the snow estimates associated with the PSD parameterizations and scattering models, a sensitivity study was done, finding that the choice of shape factor of the

gamma PSD has only a slight impact on the retrievals. As such, a value of  $\mu$  of zero, as supported by some observations, should yield reasonable estimates of snow parameters from the perspective of dual-wavelength radar retrieval.

Self-consistency of the snow retrievals has been checked using measurements of snow PSD and fall velocity acquired from the PIP during the winter of 2014 in Wallops. Among several assumptions that have been examined are conversions to particle mass spectra using different mass-size relations, scattering particle models and snow PSD. Analysis of nearly 8000 1-minute PSD measurements suggests that exponential PSD model ( $\mu$ =0) is sufficiently accurate for the dual-wavelength radar retrieval of snow bulk parameters. It also indicates that the use of either the Heymsfield or the Brandes mass-size relation yields approximately the same snow estimates. However, these findings should be viewed as preliminary because of the limited data measurements at a single location. Collections of long-term PSD data, fall velocities and information on particle mass spectra at multiple sites will provide further evaluation of the performance of the Ku- and Ka-band radar techniques. Further tests of the scattering tables will be done by comparing the accuracy of the simple particle model against scattering results from larger simulated aggregates, as these results become available.

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467	Appendix				
468		List of Symbols and Acronyms			
469	GPM:	Global Precipitation Measurement			
470	DPR:	Dual-frequency Precipitation Radar			
471	NASA:	National Aeronautics and Space Administration			
472	JAXA:	Japan Aerospace Exploration Agency			
473	Ku-band:	Frequency of 13.6 GHz			
474	Ka-band:	Frequency of 35.6 GHz			
475	PSD:	Particle Size Distribution			
476	SVI/PIP:	Snow Video Imager/Particle Image Probe			
477	DDA:	Discrete Dipole Approximation			
478	N(D):	Particle Size Distribution			
479	$N_W$ :	Scale Factor of Particle Size Distribution			
480	μ:	Shape Factor of Gamma Distribution			
481	D:	Particle Diameter			
482	$D_m$ :	Mass-weighted Diameter			
483	Λ:	Slope Parameter			
484	DFR:	Differential Frequency Ratio			
485	SWC:	SWC: Snow Water Content			
486	R <sub>S</sub> :	Equivalent Snowfall Rate			
487	LUT:	LUT: Look-up Table			
488	$Z_{Ku}$ :	Ku-band Radar Reflectivity Factor			
489	$Z_{Ka}$ :	Ka-band Radar Reflectivity Factor			
490	m(D):	Particle Mass as Function of Particle Diameter			
491	V(D):	Particle Fall Velocity			
492	$ ho_{ m w}$ :	Liquid Mass Density			
493	$ ho_{ m s}$ :	Snow Mass Density			
494	$\mathrm{D}_{min}$ :	Minimum Diameter			
495	$D_{max}$ :	Maximum Diameter			
496	f:	Frequency			
497	γ:	Aspect Ratio of Particle			
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#### References

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541

- Brandes, E., K. Ikeda, G. Zhang, M. Schoenhuber, and R. Rasmussen, 2007: A statistical and physical description of hydrometeor distributions in Colorado snowstorms using a video distrometer. *J. Appl. Meteor. Climat.*, **46**, 634-650.
- Bohm, H.P., 1989: A general equation for the terminal fall speed of solid hydrometeors. *J. Atmos. Sci.*, **46**, pp. 2419-2427.
- Botta, G., K. Aydin, and J. Verlinde, 2010: Modeling of microwave scattering from cloud ice crystal aggregates and melting aggregates: A new approach. *IEEE Geo. Rem. Sens. Lett.*, **7**, 572-576.
- Botta, G., K. Aydin, J. Verlinde, A.E. Avramov, A.S. Ackerman, A.M. Fridlind, G.M. McFarquhar and M. Wolde, 2011: Millimeter wave scattering from ice crystal aggregates: Comparing cloud model simulations with X- and Ka-band radar measurements. *J. Geophys. Res.*, **116**, D00T04.
- Botta, G., K. Aydin, and J. Verlinde, 2013: Variability in millimeter wave scattering properties of dendritic ice crystals. *J. Quant. Spectrosc. Radiat. Transf.*, **131**, 105-104.
- Bringi, V., G. Huang, V. Chandrasekar and E. Gorgucci, 2002: A methodology for estimating the parameters of a Gamma raindrop size distribution model from polarimetric radar data:
  Application to a squall-line event from the TRMM/Brazil campaign. *J. Oceanic and Atmos. Tech.*, **19**, 633-645.
- Draine, B. T., and P. J. Flatau, 1994: Discrete dipole approximation for scattering calculations. *J. Opt. Soc. Am. A.*, **11**, 1491-1499.
- Garrett, T. J., Fallgatter, C., Shkurko, K., and Howlett, D., 2012: Fall speed measurement and
   high-resolution multi-angle photography of hydrometeors in free fall. *Atmos. Meas. Tech.*, 5,
   2625-2633.
- Gorgucci, G. Scarchilli, V. Chandrasekar and V. Bringi, 2000: Measurement of mean raindrop
   shape from polarimetric radar observations. *J. Atmos. Sci.*, 57, 3406-3413.
- Gorgucci, E., G. Scarchilli, V. Chandrasekar, and V. Bringi, 2002: Estimation of raindrop size distribution parameters from polarimetric radar measurements. *J. Atmos. Sci.*, **59**, 2373-2384.
- Gravner, J., and D. Griffeath, 2009: Modeling of snow-crystal growth: A three dimensional mesoscopic approach. *Phys. Rev. E*, **79**, 011601.
- Gunn, K. L. S., and J.S. Marshall, 1958: The distribution with size of aggregate snowflakes. *J. Meteor.*, 15, 452-461.

Heymsfield, A. J., Z. Wang, and S. Matrosov, 2005: Improved radar ice water content retrieval algorithms using coincident microphysical and radar measurements. *J. Appl. Meteor.*, **44**, 1391–1412.

548

Heymsfield, A. J., and C. D. Westbrook, 2010: Advances in the estimation of ice particle fall speeds using laboratory and field measurements. *J. Atmos. Sci.*, **67**, 2469–2482.

551

Heymsfield, A. J., C. Schmitt, A. Bansemer, and C. H. Twohy, 2010: Improved representation of ice particle masses based on observations in natural clouds. *J. Atmos. Sci.*, **67**, 3303–3318.

554

Hogan, R., A. Illingworth, and H. Sauvageot, 2000: Measuring crystal size in cirrus using 35and 94-GHz radars. *J. Atmos. Oceanic Technol.*, **17**, 27-37.

557

Hou, A., G. S. Jackson, C. Kummerow, and C. M. Shepherd, 2008: Global precipitation
 measurement. Precipitation: Advances in Measurement, Estimation, and Prediction, S.
 Michaelides, Ed., Springer, 131–169.

561

Hou, A. Y., R. K. Kakar, S. Neeck, A. A. Azarbarzin, C. D. Kummerow, M. Kojima, R. Oki, K.
 Nakamura, and T. Iguchi, 2014: The global precipitation measurement mission. *Bull. American Meteor. Soc.*, 95, 701-722.

565

Huang, G., Bringi, V. N., Cifelli, R., Hudak, D. and Petersen, W. A., 2010: A Methodology to
 Derive Radar Reflectivity-Liquid Equivalent Snow Rate Relations Using C-Band Radar and
 a 2D Video Disdrometer. *J. Atmos. Oceanic Technol.*, 27, 637-651.

569

Huang, G., V.N. Bringi, D. Moisseev, W.A. Petersen, L. Bliven and D. Hudak, 2015: Use of 2D video disdrometer to derive mean density–size and Ze–SR relations: Four snow cases from
 the light precipitation validation experiment, *Atmos. Res.*, 153, 34–48.

573

Kuo, K-S, W. S. Olson, B. T. Johnson, M. Grecu, L. Tian, T. L. Clune, B. H. van Aartsen, A. J.
 Heymsfield, L. Liao, and R. Meneghini, 2016: The microwave radiative properties of falling snow derived from realistic ice particle models. Part I: An extensive database of simulated pristine crystals and aggregate particles, and their scattering properties. *J. Appl. Meteorol. Climatol.*, 55, 691-708.

579

Liao, L., R. Meneghini, H. Nowell and G. Liu, 2013:Scattering computations of snow aggregates
 from simple geometrical particle models. *IEEE J. Selected Topics in Earth Obs. and Remote Sens.*, 6, 1409-1417.

583

Liu, G., 2004: Approximation of single scattering properties of ice and snow particles for high microwave frequencies. *J. Atmos. Sci.*, **61**, 2441-2456.

586

Liu, G., 2008: A database of microwave single-scattering properties for non spherical ice particles. *Bull. Am. Meteorol. Soc.*, **89**, 1563-1570.

Locatelli, J., and P. Hobbs, 1974: Fall speeds and masses of solid precipitation particles. *J. Geophys. Res.*, 79, 2185-2197.

592

Magono C., and T. Nakamura, 1965: Aerodynamic studies of falling snowflakes, *J. Meteorol. Soc. Jpn.*, 43, 139–147.

595

596 Matrosov, S. Y., 1998: A dual-wavelength radar method to measure snowfall rate. *J. Appl. Meteor.*, **37**, 1510–1521.

598

Matrosov, S. Y., A. J. Heymsfield, and Z. Wang, 2005: Dual-frequency ratio of non-spherical atmospheric hydrometeors. *Geophys. Res. Lett.*, **32**, L13816, doi:10.1029/2005GL023210.

601

Mitchell, D. L., R. Zhang, and R. L. Pitter, 1990: Mass-dimensional relations for ice particles and the influence of riming on snowfall rates. *J. Appl. Meteor.*, **29**, 153-163.

604

Nakaya, U., 1954: Snow Crystals: Natural and Artificial. Harvard University Press, 510pp.

606

Newman, A., P. A. Kucera, and L. Bliven, 2009: Presenting the snow video imager. *J. Atmos. Ocean.* Technol., **26**, 167-179.

609

Nowell, H., G. Liu, and R. Honeyager, 2013: Modeling the microwave single-scattering properties of aggregate snowflakes. *J. Geophys. Res. Atmos.*, **118**, 7873–7885. doi:10.1002/jgrd.50620.

613

Petty, G. W., and W. Huang, 2010: Microwave backscattering and extinction by soft ice spheres and complex snow aggregates. *J. Atmos. Sci.*, **67**, pp. 769-787.

616

Seto, S, T. Iguchi, and T. Oki, 2013: The basic performance of a precipitation retrieval algorithm for the global precipitation measurement mission's single/dual-frequency radar measurements. *IEEE Trans. Geosci. Remote Sens.*, **51**, 5239–5251.

620

Szyrmer, W., and I. Zawadzki, 2014a: Snow studies. Part III: Theoretical derivations for the ensemble retrieval of snow microphysics from dual-wavelength vertically pointing radars. *J. Atmos. Sci.*, **71**, 1158–1170.

624

Szyrmer, W., and I. Zawadzki, 2014b: Snow studies. Part IV: Ensemble retrieval of snow microphysics from dual-wavelength vertically pointing radars. *J. Atmos. Sci.*, **71**, 1172–1186.

627

Wang, Z., G. M. Heymsfield, L. Li, and A. J. Heymsfield, 2005: Retrieving optically thick ice
 cloud microphysical properties by using airborne dual-wavelength radar measurements. *J. Geophys. Res.*, 110, D19201, doi:10.1029/2005JD005969.

631

Weinman, J., and M.-J. Kim, 2007: A simple model of the millimeter-wave scattering parameters of randomly oriented aggregates of finite cylindrical ice hydrometeors. *J. Atmos. Sci.*, **64**, 634-644.

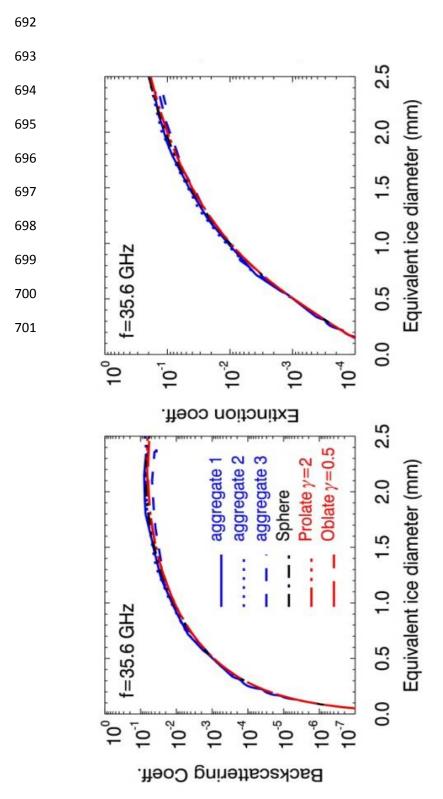
Zikmunda, J., and G. Vali, 1972: Fall patterns and fall velocities of rimed ice crystals. *J. Atmos. Sci.*, 29, 1334-1347.

Table 1 Snow events during winter of 2014 in Wallops Island, Virginia

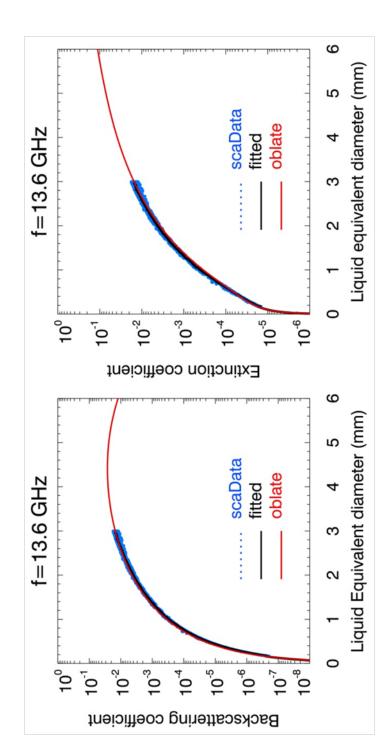
Events	Start Time (UTC)	End Time (UTC)	Accumulation (mm)	Mean Temperature ( <sup>0</sup> C)
1	JAN0305:09	JAN03 11:30	46.41	-1.4
2	JAN21 22:05	JAN22 10:03	5.52	-5.4
3	JAN28 20:41	JAN29 12:40	62.12	-9.5
4	FEB14 01:58	FEB14 05:12	6.37	1.9
5	FEB15 20:41	FEB15 23:23	0.76	2.2
6	MAR03 14:40	MAR03 22:00	34.15	-4.4
7	MAR17 08:04	MAR17 20:53	20.93	0.8
8	MAR25 18:50	MAR26 06:13	47.40	1.0

- Figure captions:
- Fig. 1 Comparisons of backscattering (left) and extinction (right) coefficients of 3 snow
- aggregates with the results from the sphere, oblate and prolate spheroids at a frequency of 35.6
- 645 GHz in terms of equivalent ice diameter. A constant snow density of 0.2 g/cm<sup>3</sup> is assumed for all
- spherical and spheroidal particle models. The oblate and prolate spheroids are randomly oriented
- with aspect ratios ( $\gamma$ ) of 0.5 and 2, respectively.
- Fig.2 Backscattering (left) and extinction (right) coefficients from the scattering database (blue
- dots), simple scattering model (red curves), which is the randomly-oriented oblate spheroid with
- a constant effective density of 0.2 g/cm<sup>3</sup> and an aspect ratio of 0.7, and scattering-database-
- simple-model-combined results (black curves), also referred to as "fitted", at Ku band. The mean
- values of the scattering results are used for the combined results over the data range.
- Fig.3 Backscattering (left) and extinction (right) coefficients from the scattering database (blue
- dots), simple scattering model (red curves), which is the randomly-oriented oblate spheroid with
- a constant effective density of 0.2 g/cm<sup>3</sup> and an aspect ratio of 0.7, and scattering-database-
- simple-model-combined results (black curves), also referred to as "fitted", at Ka band. The mean
- values of the scattering results are used for the combined results over the data range.
- Fig.4 The differential frequency ratio (DFR= $10LOG_{10}(Z_{Ku}/Z_{Ka})$ ) as a function of equivalent-
- 659 liquid median mass diameter D<sub>m</sub>. (Left): DFR-D<sub>m</sub> relations are plotted with several effective
- snow densities ( $\rho_s$ ) from 0.1 to 0.4 g/cm<sup>3</sup> as the shape factor ( $\mu$ ) of the gamma PSD is set to zero
- and the aspect ratio ( $\gamma$ ) of the oblate spheroid particles is set to 0.7. (Middle): DFR-D<sub>m</sub> relations
- are plotted with the aspect ratios of 0.5, 0.7 and 1 at  $\rho_s$ =0.2 g/cm<sup>3</sup> and  $\mu$ =0. (Right): DFR-D<sub>m</sub>
- relations are plotted with the values of  $\mu$  of 0, 3 and 6 at  $\rho_s$ =0.2 g/cm<sup>3</sup> and  $\gamma$ =0.7.
- Fig. 5 The retrieval look-up tables that show the snow water content (SWC) (top-left) and
- equivalent snowfall rate (R<sub>S</sub>) (top-right), both of which are normalized by the Ku-band radar
- reflectivity factor ( $Z_{Ku}$ ), as a function of the DFR, defined by  $10LOG_{10}(Z_{Ku}/Z_{Ka})$ , for several
- effective snow densities ( $\rho_s$ ) with the values from 0.05 to 0.5 g/cm<sup>3</sup>. The liquid equivalent
- median mass diameter  $D_m$  (bottom-left) and the PSD scale parameter  $N_w$  normalized by  $Z_{Ku}$
- 669 (bottom-right) are also plotted in terms of DFR.
- Fig.6 Flowchart of computing radar parameters and snow size and bulk properties.
- Fig. 7 The look-up tables used for the retrieval of SWC (left) and  $R_S$  (right) with  $\mu$  of 0, 3 and 6,
- respectively, as computed from the single scattering tables depicted in Figs.2-3.
- Fig. 8 Example of a segment of the PSD measurements (1000 minutes) in time series taken from
- 8 snow events during winter of 2014 at the NASA Wallops Flight Facility using the SVI/PIP.
- The particle size spectra (mm<sup>-1</sup> m<sup>-3</sup>), shown in the color scale, are given in the top panel while
- equivalent snow fall rate and actual median mass diameter are displayed in the middle and

677 bottom panels, respectively. The PSD data, obtained by averaging the measurements over 1minute integration time, are merged from all the snow events into one data file with consecutive 678 679 time. Fig. 9 The snow water content (SWC) (top row) and equivalent snowfall rate (R<sub>S</sub>) (bottom row), 680 both of which are normalized by the Ku-band radar reflectivity factor (Z<sub>Ku</sub>), as a function of the 681 DFR, defined by  $10LOG_{10}(Z_{Ku}/Z_{Ka})$ , for several effective snow densities ( $\rho_s$ ) with the values 682 from 0.05 to 0.5 g/cm<sup>3</sup>. The scatter plots (red dots) are the results derived from the measured 683 PSD that were collected by the SVP/PIP from 8 snow events in the winter of 2014 at the NASA 684 Wallops Flight Facility. Two empirical density-size relationships reported by Brandes et al. 685 (2007) and Heymsfield et al. (2010) are used in converting the measured PSD to the snow mass 686 spectra, and the results from their respective relations are shown in the left and right panels. 687 Fig.10 Flowchart detailing the procedures of obtaining radar reflectivities and snow parameters. 688 689 690

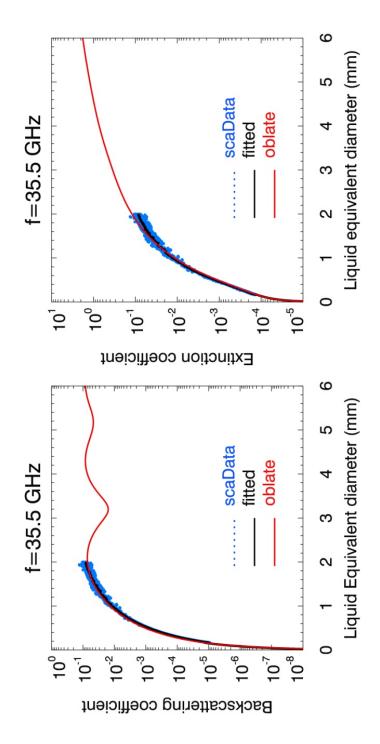


constant snow density of 0.2 g/cm<sup>3</sup> is assumed for all spherical and spheroidal particle models. The oblate and prolate Fig.1 Comparisons of backscattering (left) and extinction (right) coefficients of 3 snow aggregates with the results from the sphere, oblate and prolate spheroids at a frequency of 35.6 GHz in terms of equivalent ice diameter. A spheroids are randomly oriented with aspect ratios ( $\gamma$ ) of 0.5 and 2, respectively.

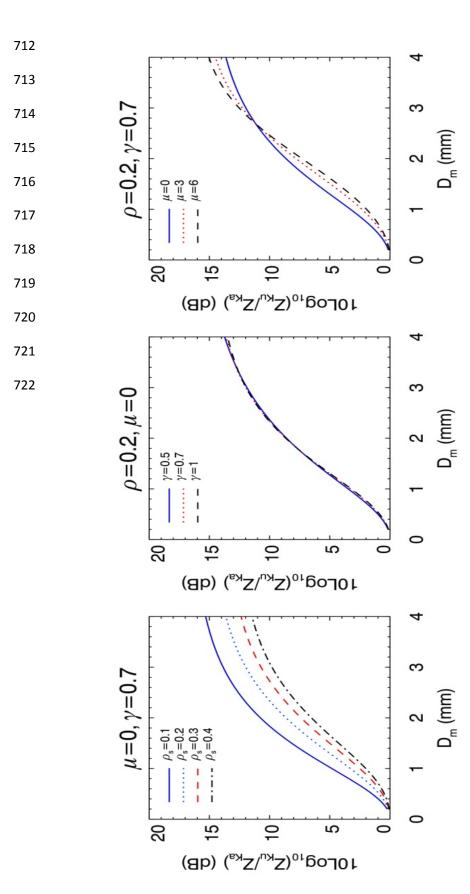


simple scattering model (red curves), which is the randomly-oriented oblate spheroid with a constant effective density referred to as "fitted". The mean values of the scattering results are used for the combined results over the data range of 0.2 g/cm<sup>3</sup> and an aspect ratio of 0.7, and scattering-database-simple-model-combined results (black curves), also Fig.2 Backscattering (left) and extinction (right) coefficients at Ku-band from the scattering database (blue dots),





scattering model (red curves), which is the randomly-oriented oblate spheroid with a constant effective density of 0.2 Fig.3 Backscattering (left) and extinction (right) coefficients from the scattering database (blue dots), simple g/cm<sup>3</sup> and an aspect ratio of 0.7, and scattering-database-



D<sub>m</sub>. (Left): DFR-D<sub>m</sub> relations are plotted with several effective snow densities (ρ<sub>s</sub>) from 0.1 to 0.4 g/cm<sup>3</sup> as the shape factor  $(\mu)$  of the gamma PSD is set to zero and the aspect ratio  $(\gamma)$  of the oblate spheroid particles is set to 0.7. (Middle): DFR-D<sub>m</sub> relations are plotted with the aspect ratios of 0.5, 0.7 and 1 at  $\rho_s$ =0.2 g/cm<sup>3</sup> and  $\mu$ =0. (Right): DFR-D<sub>m</sub> relations are plotted Fig.4 The differential frequency ratio (DFR= $10LOG_{10}(Z_{Ku}/Z_{Ka})$ ) as a function of equivalent-liquid median mass diameter with the values of  $\mu$  of 0, 3 and 6 at  $\rho_s=0.2$  g/cm<sup>3</sup> and  $\gamma=0.7$ .

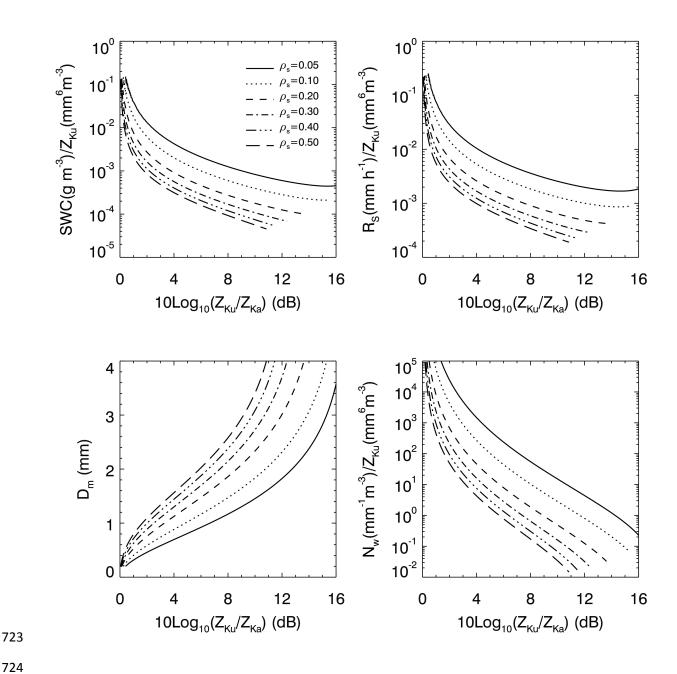


Fig.5 The retrieval look-up tables that show the snow water content (SWC) (top-left) and equivalent snowfall rate ( $R_S$ ) (top-right), both of which are normalized by the Ku-band radar reflectivity factor ( $Z_{Ku}$ ), as a function of the DFR, defined by  $10LOG_{10}(Z_{Ku}/Z_{Ka})$ , for several effective snow densities ( $\rho_s$ ) with the values from 0.05 to 0.5 g/cm<sup>3</sup>. The liquid equivalent median mass diameter  $D_m$  (bottom-left) and the PSD scale parameter  $N_w$  normalized by  $Z_{Ku}$  (bottom-right) are also plotted in terms of DFR.

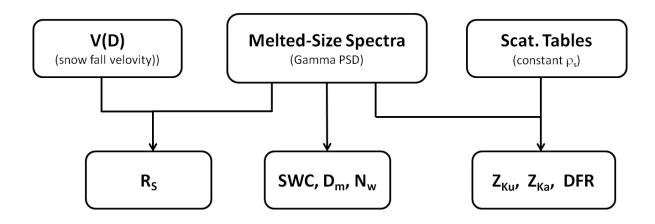


Fig.6 Flowchart of computing radar parameters and snow size and bulk properties.

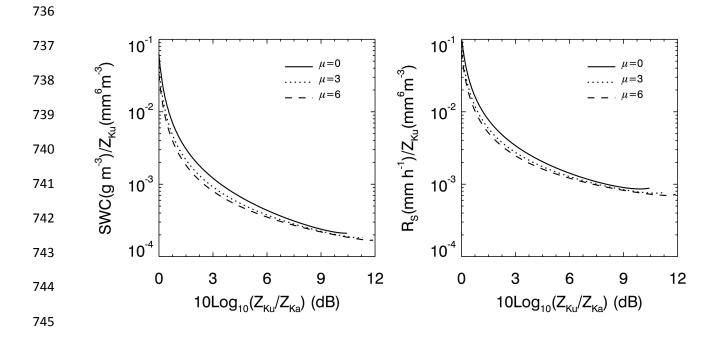


Fig.7 The look-up tables used for the retrieval of SWC (left) and  $R_S$  (right) with  $\mu$  of 0, 3 and 6, respectively, as computed from the single scattering tables depicted in Figs.2-3.

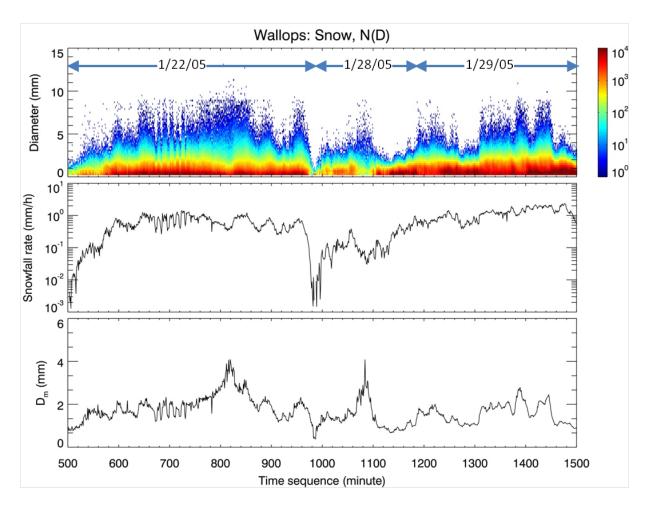


Fig.8 Example of a segment of the PSD data, obtained by averaging the measurements over 1-minute integration time, in time series taken from 8 snow events during winter of 2014 at the NASA Wallops Flight Facility using the SVI/PIP. The particle size spectra (mm<sup>-1</sup> m<sup>-3</sup>), shown in the color scale, are given in the top panel while equivalent snow fall rate and actual median mass diameter are displayed in the middle and bottom panels, respectively. The dates provided in the top panel correspond to the snow events displayed.

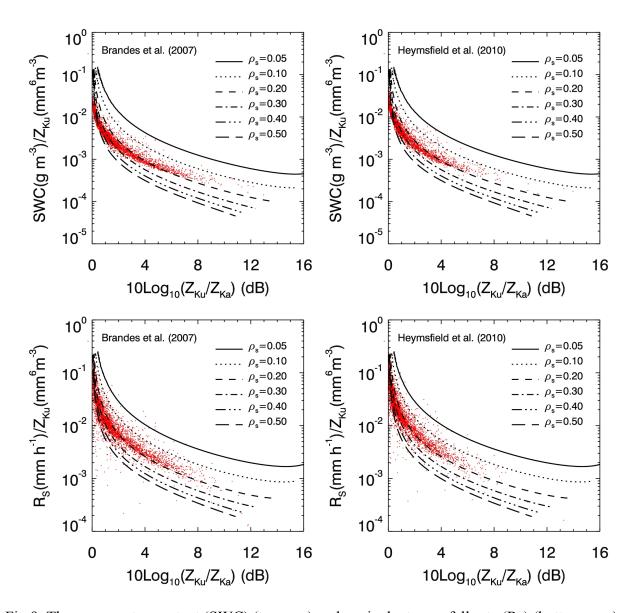


Fig.9 The snow water content (SWC) (top row) and equivalent snowfall rate ( $R_S$ ) (bottom row), both of which are normalized by the Ku-band radar reflectivity factor ( $Z_{Ku}$ ), as a function of the DFR, defined by  $10LOG_{10}(Z_{Ku}/Z_{Ka})$ , for several effective snow densities ( $\rho_s$ ) with the values from 0.05 to 0.5 g/cm<sup>3</sup>. The scatter plots (red dots) are the results derived from the measured PSD that were collected by the SVP/PIP from 8 snow events in the winter of 2014 at the NASA Wallops Flight Facility. Two empirical density-size relationships reported by Brandes et al. (2007) and Heymsfield et al. (2010) are used in converting the measured PSD to the snow mass spectra, and the results from their respective relations are shown in the left and right panels.

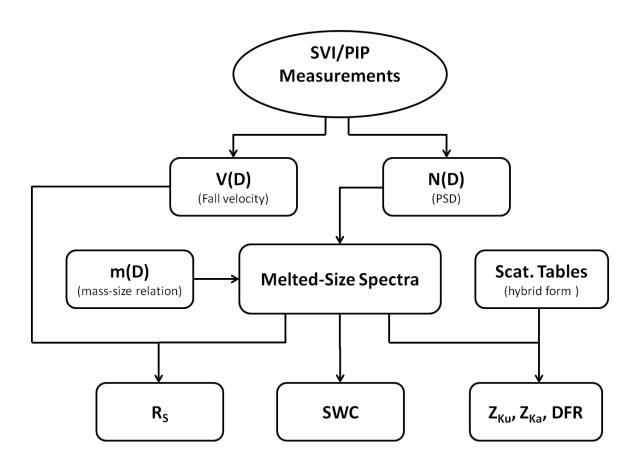


Fig.10 Flowchart detailing the procedures of obtaining radar reflectivities and snow parameters.